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DETERMINATION OF THE REGION OF STABLE AND RELIABLE OPERATION OF EQUILIBRIUM TWO-PHASE TRANSPIRATION COOLING SYSTEM

A. V. LUIKOV,† L. L. VASILIEV and V. A. MAYOROV
Heat & Mass Transfer Institute, Minsk, U.S.S.R.

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Abstract—With the use of the stability conditions and the absence of homogeneous wall burnout analytical expressions are derived for the region of parameters with a change inside which the system of two-phase porous cooling is stable and the heated surface temperature does not exceed the limit for reliable use of a porous material. The system stability is found to depend on the substantial number of parameters, the kind of the coolant being one of them.

The stability condition imposes very strict limitations upon the characteristics of the system which is accounted for by a sharp decrease in the coolant flow rate in case of recession of the evaporation region from the external surface inside the plate. It is obvious from the comparison of the results obtained and the parameters of the experimental installations described in the literature, that neglect of the above requirements is indeed one of the main reasons for the instability of two-phase transpiration cooling of a homogeneous wall, with water being used as a coolant.

NOMENCLATURE

G, g, dimensional and non-dimensional coolant mass flow rates;
l, non-dimensional coordinate of the phase conversion region;
δ, wall thickness;
P_0, delivery pressure;
P_1, ambient pressure;
q, external heat flux density;
α, β, viscous and inertial resistance coefficients;
λ_2, effective thermal conductivity of vapour section;
μ, dynamic viscosity;
ρ, density;
c, heat capacity;
i, enthalpy;
Re, Reynolds number of a coolant flow;
ν, kinematic viscosity.

Superscripts

$'$, physical properties of liquid in a saturated state;
$''$, physical properties of vapour in a saturated state;
$*$, parameters at the stability boundary;
$**$, parameters at the reliability boundary.

INTRODUCTION

IN PAPER [1] the laws of cooling with coolant phase conversion inside a homogeneous flat porous wall subjected to surface heating are analysed. Characteristics are plotted which make it possible not only to draw a conclusion about the stability of the system but also to define permissible disturbances of the controlling parameters in a stable system. Still there is an uncertainty as to the ways of solving the main problem which arises in design of the cooling system: what kind of thermophysical and structural characteristics should the porous wall possess that the system with the known characteristic parameters (density of external heat flux, ambient pressure, initial coolant temperature) be not only stable but its stability be preserved in case of some disturbances of these parameters. The external surface temperature should not exceed the limit for reliable use of porous materials. The permissible range of variations of the characteristic parameters depends on the particular construction of the system and is supposed to be known.

The formulated problem is nothing but a problem on determination of the region of stability and reliability. This is the region where the variation of parameters do not disturb stable and reliable operation of the cooling system. The parameters required can be found by their arbitrary selection and subsequent check by plotting all the static characteristics which is to be repeated until the necessary requirements are met. Excessive amount of calculation is quite obvious in this case.

ANALYTICAL DETERMINATION OF THE STABILITY REGION OF THE TWO-PHASE TRANSPIRATION COOLING SYSTEM

Based on the stability condition derived in the earlier [1] paper, an analytical solution of the problem on determination of the stability region is suggested below. Here holds the condition to be achieved from the analysis of the thermal characteristic: the system of two-phase transpiration cooling is stable provided that the working point belongs to the falling section in the curve of density of the supplied external heat flux q vs non-dimensional coordinate l of the surface of equilibrium phase conversion inside the wall

$$\frac{dq}{dl} < 0. \quad (1)$$

† Deceased.

This should be supplemented with the analytical expression of the fact that the working point belongs to the thermal characteristics (the condition of equilibrium phase conversion)

$$q = (i_e'' - c_m T_\infty)G \exp\left[\frac{G\delta c''}{\lambda_2'}(1-l)\right]. \quad (2)$$

Here the coolant flow rate G and, due to an increase in the pressure in the evaporation region recessing into the plate, the enthalpy i_e'' of the generated dry saturated vapour depend on the coordinate of the equilibrium phase transition surface.

The above relations hold in the stability region which can be found by defining its boundary, where inequality (1) transits into the equality

$$\frac{dq}{dl} = 0. \quad (3)$$

Upon substituting (2) into (3) and making the appropriate transformations we have the relation between the parameters at the stability boundary

$$\frac{\lambda_2}{G\delta c''} = \frac{\left[(l-1)\frac{1}{g}\frac{dg}{dl}+1\right]}{\left[\frac{1}{g}\frac{dg}{dl}+\frac{1}{i_e''-c_m T_\infty}\frac{di_e''}{dl}\right]}. \quad (4)$$

The expression incorporates the thickness of the porous wall δ, the effective thermal conductivity λ_2 of the porous material–vapour section, coolant flow rate

$$G = gG_1 \quad (5)$$

equal to the product of the viscous coolant flow rate G_1 (Darcy flow) under the effect of the pressure drop $P_0 - P_1$ on the plate

$$G = \frac{P_0 - P_1}{\delta v'\alpha} \quad (6)$$

and non-dimensional flow rate

$$g = \frac{-1+\sqrt{\left(1+4Re\frac{n}{m^2}\right)}}{2Re\frac{n}{m}}. \quad (7)$$

The dependence of the coolant flow rate on the position of the phase transition surface is accounted for in formula (7) by auxiliary complexes

$$m = \left[l+\frac{v''}{v'}(1-l)\right]; \quad n = \left[l+\frac{\rho'}{\rho''}(1-l)\right]. \quad (8)$$

Moreover the flow rate depends on the flow pattern described by the Reynolds number of the flow

$$Re = \frac{G_1\beta/\alpha}{\mu'} = \frac{(P_0-P_1)\beta/\alpha}{\delta v'\alpha\mu'} \quad (9)$$

and on the physical properties μ', v', ρ' of the liquid and of the vapour phases of the coolant, which are calculated in a state of saturation with the pressure being equal to the known ambient pressure P_1. Equation (4) incorporates the heat capacity of saturated vapour c'' and mean heat capacity of liquid c_m from 0°C to the initial coolant temperature T_∞. The characteristic dimension β/α of the porous structure is a ratio of inertial β and viscous α drag coefficients.

From relation (4) it follows that the system parameters at the stability boundary depend considerably on the relative change in the coolant flow rate

$$\frac{1}{g}\frac{dg}{dl}$$

when phase conversion surface is moving inside the plate. The expression for calculation of this derivative

$$\frac{1}{g}\frac{dg}{dl} = \frac{\frac{2n}{m}\left(\frac{v''}{v'}-1\right)+\left(\frac{\rho'}{\rho''}-1\right)\left[-1+\sqrt{\left(1+4Re\frac{n}{m^2}\right)}\right]}{2n\sqrt{\left(1+4Re\frac{n}{m^2}\right)}} \quad (10)$$

is obtained from formula (7) and non-dimensional equation of liquid motion in porous material

$$1 = gm + g^2 nRe \quad (11)$$

for which relation (7) is the solution.

To reduce the summand

$$\frac{1}{i_e''-c_m T_\infty}\frac{di_e''}{dl}$$

in (4) to an analytical form, we linearize the dependence of the enthalpy of dry saturated vapour on the saturation pressure by a segment of the tangent drawn at the point where the pressure equals the ambient pressure

$$\left.\frac{di''}{dP}\right|_{P=P_e} = \left.\frac{di''}{dP}\right|_{P=P_1}. \quad (12)$$

The variation of the saturated vapour enthalpy at recession of the evaporation region is proportional to the difference of pressure P_e in the equilibrium phase transition region and ambient pressure P_1

$$i_e'' - i_{e=1}'' = \left.\frac{di}{dP}\right|_{P_1}(P_e - P_1). \quad (13)$$

Upon expressing the coolant enthalpy variation $i_e'' - c_m T_\infty$ in the form

$$(i_e'' - c_m T) = (i_e'' - i_{e=1}'') + (i_{e=1}'' - c_m T_\infty) \quad (14)$$

and accounting for (12) and (13), and making some transformations, we arrive at

$$\frac{1}{i_e''-c_m T_\infty}\cdot\frac{di_e''}{dl} = \frac{\frac{1}{P_e-P_1}\cdot\frac{dP_e}{dl}}{\left[1+\frac{i_{e=1}''-c_m T_\infty}{(P_e-P_1)\left.\frac{di''}{dP}\right|_{P_1}}\right]} \quad (15)$$

The pressure drop $P_e - P_1$ at the vapour section of the coolant flow is to be calculated in fractions of the total pressure drop $P_0 - P_1$ on the porous plate from the relation

$$P_e - P_1 = (P_0 - P_1)(l - gl - g^2 lRe) \quad (16)$$

and then

$$\frac{dP_e}{dl} = (P_0 - P_1)\left[g + g^2 Re + l\frac{1}{g}\frac{dg}{dl}(g + 2g^2 Re)\right]. \quad (17)$$

Substituting expressions (13)–(17) and (5)–(10) into condition (4) we obtain one of the equations to determine the parameters at the stability boundary of the two-phase transpiration cooling system. Since none of the equations (5)–(10), (13)–(17) incorporates thermal conductivity of vapour λ_2, it is only natural to solve equation (4) with respect to its boundary value λ_2^*.

Substitution of the expression for the boundary value λ_2^* as well as relations to calculate the coolant flow rate (5) and (9) and the enthalpy variation (14), (13), (16), (7)–(9) into the condition of equilibrium transition (2) makes it possible to write also an equation for determining the heat flux q^* corresponding to the state at the stability boundary.

According to [2] in a wide range of pressures from 1 to 120 bars the enthalpy of saturated water vapour remains constant within 4·5 per cent, hence

$$\frac{1}{i_e'' - c_m T_\infty}\frac{di_e}{dl} = 0.$$

With water as a coolant we obtain the following parametric equations for the stability boundary

$$\lambda_2^* = \frac{\delta c'' \mu'}{\beta/\alpha}\frac{m}{2n}\left[-1 + \sqrt{\left(1 + 4Re\frac{n}{m^2}\right)}\right](l-1)\left\{1 + \frac{2n\sqrt{\left(1+4Re\frac{n}{m^2}\right)}}{(l-1)\left[\left(\frac{v''}{v'} - 1\right)\frac{2n}{m} + \left(\frac{\rho'}{\rho''} - 1\right)\left(-1 + \sqrt{\left(1 + 4Re\frac{n}{m^2}\right)}\right)\right]}\right\} \quad (18)$$

$$q^* = (i - c_m T_\infty)\frac{\mu'}{\beta/\alpha}\frac{m}{2n}\left[-1 + \sqrt{\left(1 + 4Re\frac{n}{m^2}\right)}\right]$$

$$\times \exp\left\{\frac{-1}{1 + \frac{2n\sqrt{\left(1+4Re\frac{n}{m^2}\right)}}{(l-1)\left[\left(\frac{v''}{v'} - 1\right)\frac{2n}{m} + \left(\frac{\rho'}{\rho''} - 1\right)\left(-1 + \sqrt{\left(1 + 4Re\frac{n}{m^2}\right)}\right)\right]}}\right\}. \quad (19)$$

For systems with other coolants the stability boundary is described in a tedious but, which is most important, analytical form. Here the set of parameters incorporated into a brief parametric representation of the stability boundary

$$\lambda_2^* = \lambda_2^*(l, Re, P_1, \beta/\alpha, \delta, \text{ kind of coolant}) \quad (20)$$

$$q^* = q^*(l, Re, P_1, \beta/\alpha, T_\infty, \text{kind of coolant}) \quad (21)$$

remains the same.

It should be noted that the accuracy of the assumptions of constant physical properties of both coolant phases and on their equality to relevant values in a saturated state, as well as on the linear dependence of the saturated steam enthalpy on saturation pressure increases as the coolant evaporation zone approaches the external surface, when $l \to 1{\cdot}0$ and $P_e \to P_1$. Hence, in this case the accuracy of calculation of the stability boundary by equations (20)–(21) or (18)–(19) increases as well.

DISCUSSION OF RESULTS

Equations (20)–(21) or their particular case (18)–(19) present a parametric way of determining the stability boundary in coordinates $\lambda_2 - q$. The effect of the parameters $\beta/\alpha, \delta, T_\infty$ upon the boundary values of λ_2^*, q^* is obvious. The boundary thermal conductivity of vapour section increases in proportion with the plate thickness δ, while the growth of the initial temperature T_∞ reduces the limit heat flux q^* in a stable system. At the same time it is not so easy to estimate the contribution of the parameters l, Re, P_1. Therefore we shall choose the real values of the parameters β/α, δ, T_∞, fix P_1 and plot the stability boundary changing consecutively one of the remaining parameters l, Re.

The following parameters are assumed as initial ones: initial temperature of the water coolant $T_\infty = 20°C$; ambient pressure $P_1 = 10$ bar, wall thickness $\delta = 5$ mm; porosity $\prod = 0{\cdot}2$. Viscous and inertial drag coefficients $\alpha = 3{\cdot}5 \times 10^{13} m^{-2}$ and $\beta = 1{\cdot}2 \times 10^8 m^{-1}$, respectively, and characteristic dimension of porous structure $\beta/\alpha = 3{\cdot}44 \times 10^{-6}$ m are calculated on the basis of the experimental data for a plate of similar thickness and porosity sintered of stainless steel powder [3].

In Fig. 1 curve 1 is the stability boundary at constant Reynolds numbers of the coolant flow $Re = 0{\cdot}1$. Numbers along the curve stand for the values of

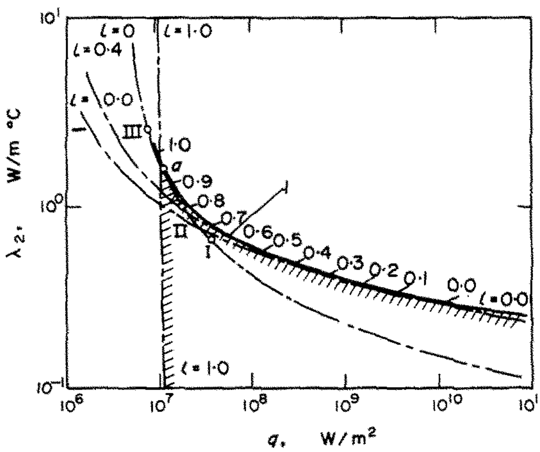

FIG. 1. Stability region of equilibrium two-phase transpiration cooling system at $Re = 0{\cdot}1$.

the coordinate of the phase transition surface in the interval $l \in [0{\cdot}1]$ relevant to the marked points. Constant pressure drop $P_0 - P_l$ bars corresponds to the indicated Reynolds number.

The plotted boundary, however, does not provide any idea about the stability region, which should be filled as follows. We shall consider a system in which the phase transition surface has the coordinate $l = 0{\cdot}95$. If the system is in a state close to the stability boundary, its parameters are represented by the point a. For the given conditions the equilibrium phase transition in the region $l = 0{\cdot}95$ can be ensured with the parameters λ_2^*, q^* having not only boundary values but also some other ones, provided that the equilibrium condition (2) is satisfied. A set of such parameters makes a dot–dash line of the equilibrium phase transformation $l = 0{\cdot}95$ passing via the point a. Plotting of thermal characteristics shows that the parameters λ_2, q relevant to the points of this curve in the left-hand upward direction from the boundary value place this cooling system among unstable ones. It is of interest to note that the point a is a point of contact of the dot-and-dash line $l = 0{\cdot}95$ and the stability boundary.

So, the stability region is filled with the family of dot-and-dash equilibrium phase transition line continuations $l = \text{const}$ in the right-hand downward direction from the points of their contact with the stability boundary

$$\lambda_2 < \lambda_2^* \qquad q > q^*. \tag{22}$$

This region is bounded with the boundary and continuations of dash-and-dot lines $l = 1{\cdot}0$ and $l = 0{\cdot}0$. The stability region in Fig. 1 is hatched.

It is also essential to note that the lines $l = \text{const}$ partially pass through the hatched region to the point of contact with the stability boundary and cross the lines $l = \text{const}$ which have already been in contact with it. No more than two lines intersect at the same point. For example, lines $l = 0{\cdot}4$ and $l = 0{\cdot}95$ cross each other at point II. A system with such parameters is stable if the coolant evaporation zone has the coordinate $l = 0{\cdot}95$, and it is unstable if $l = 0{\cdot}4$. This and also that phase transition in the region $l = 0{\cdot}95$ in a system with the parameters presented by point I is stable, and by point III is unstable, is obvious from the static characteristics of these systems plotted in the previous paper.

The Reynolds number $Re = 0{\cdot}1$ indicates the coolant flow pattern with dominating viscous drag. The stability boundaries in the transient flow $Re = 1{\cdot}0$ and with dominating inertial drag $Re = 31{\cdot}6$ are presented in Figs. 2 and 3. The increase of the inertial drag in a transient flow results in an extremum point of the stability boundary, $Re = \text{const}$. The extremum point first appears at $l = 1{\cdot}0$ and gradually changes the correlation between two boundary sections. One of them $l \in [l^v, 0]$ has a characteristic shape of a viscous flow pattern, the other section $l \in [1, l^v]$ of the inertial one. The analysis shows that the extremum point parameters satisfy simultaneously three conditions, that is, the phase transition equilibrium condition (2); the state at

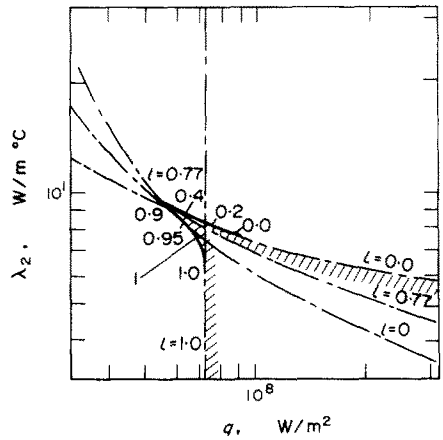

FIG. 2. Stability region of equilibrium two-phase transpiration cooling system at $Re = 1{\cdot}0$.

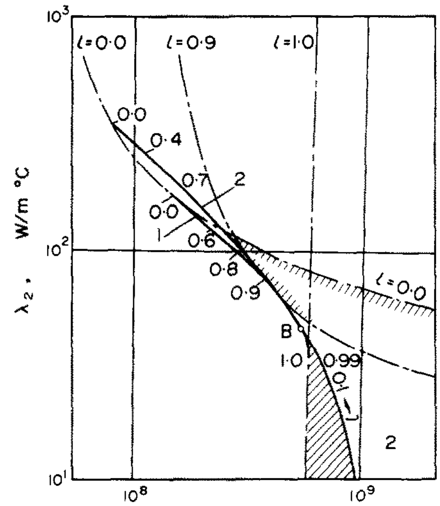

FIG. 3. Region of stable and reliable operation of equilibrium two-phase transpiration cooling system at $Re = 31{\cdot}6$: 1, stability boundary; 2, reliability boundary.

the stability boundary (3); bend of thermal characteristic

$$\frac{d^2q}{dl^2} = 0.$$

If we plot a set of stability boundaries $Re = \text{const}$ and connect the points with equal values of the equilibrium phase transition coordinate by the lines $l = \text{const}$, we obtain two families of curves presented in Fig. 4. The ranges of viscous, transient and inertial flow patterns in a porous structure can easily be observed by the shape of the curves of these families.

The lines $l = \text{const}$ for viscous and inertial flow patterns are straight and the shape of the stability boundary $Re = \text{const}$ does not depend on the Reynolds number. This follows from its analytical representation at limit transitions in equations (18) and (19) for the

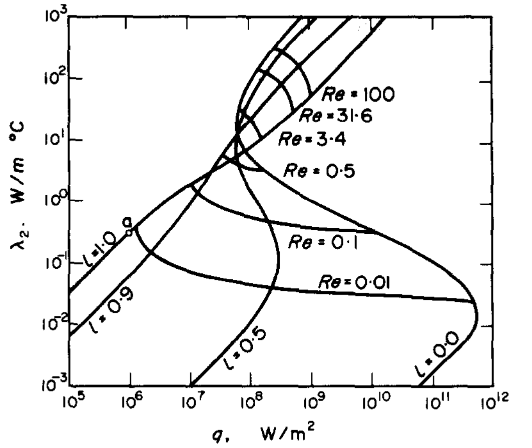

FIG. 4. Family curves Re = const and l = const at ambient pressure $P_1 = 10$ bar.

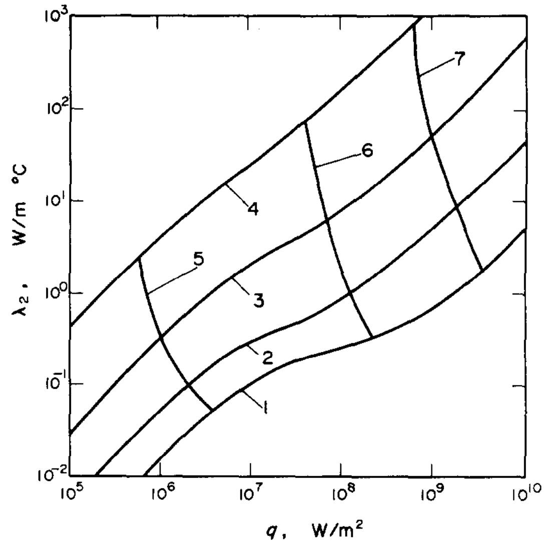

FIG. 5. Boundary parameters of equilibrium two-phase transpiration cooling system with evaporation from external surface vs ambient pressure: 1, $P_1 = 0{\cdot}1$ bar; 2, $P_1 = 1{\cdot}0$ bar; 3, $P_1 = 10$ bar; 4, $P_1 = 100$ bar; 5, $Re = 0{\cdot}01$ bar; 6, $Re = 1{\cdot}0$; 7, $Re = 100$.

viscous $Re \to 0$

$$\lambda_2^* = q^* \frac{\delta c''}{(i'' - c_m T_\infty)\left(\frac{v''}{v'} - 1\right)} \exp\left[(l-1)\left(\frac{v''}{v'} - 1\right)\right] \quad (23)$$

and inertial $Re \to \infty$ flow patterns

$$\lambda_2^* = q^* \frac{\delta c''}{(i'' - c_m T_\infty)}(l-1) \times \left[1 + \frac{2n}{(l-1)\left(\frac{\rho'}{\rho''} - 1\right)}\right] \times \exp\left[\frac{1}{1 + \frac{2n}{(l-1)\left(\frac{\rho'}{\rho''} - 1\right)}}\right] \quad (24)$$

It is also interesting to note that in case of stability boundary construction the use of relevant transpiration cooling criterion rather than of thermal conductivity of the vapour section allows very simple form of equation (18) for the viscous flow pattern

$$K_2^* = \frac{G\delta c''}{\lambda_2^*} = \left(\frac{v''}{v'} - 1\right). \quad (25)$$

The ambient pressure P_1 has a substantial effect on the boundary parameters of the system since the physical properties of both coolant phases, incorporated in formulae (18)–(19) vary sharply with the saturation pressure. Some figures of the type of Fig. 4 for different ambient pressures would provide a most comprehensive picture of the relevant variation of the stability region in the cooling system. The main features, however, can easily be revealed even without such bulky procedure. It is enough to follow the variation of the most characteristic line $l = 1{\cdot}0$. Curves 1–4 in Fig. 5 show the general trend of the boundary value of the vapour effective thermal conductivity to grow with pressure in the system during the phase transition on the external surface. Moreover, the curve $l = 1{\cdot}0$ tends to straighten, i.e. the difference between its asymptotes for the viscous and inertial flow patterns decreases. Curves 5–7 show variation of the boundary parameters with pressure, the Reynolds number of the coolant flow being constant.

The quantitative aspect of the results presented in Fig. 4 deserves special attention. For a viscous flow pattern the system is characterized by the maximal vapour thermal conductivity with evaporation on the external surface. For this case the relation derived from (23) for this case

$$\lambda_2^* = q^* \frac{\delta c''}{(i'' - c_m T_\infty)\left(\frac{v''}{v'} - 1\right)} \quad (26)$$

makes it possible to find, by the known heat flux, the ultimate thermal conductivity of porous two-phase medium, which when being exceeded makes it entirely impossible for the system to reach any stability. Formula (26) as considered holds within the range of heat fluxes up to $3{\cdot}0 \times 10^6$ W/m^2. And so it turns out, that for the external heat flux density $q = 10^6$ W/m^2, the porous medium thermal conductivity should be less than a very small limit value $\lambda_2 < \lambda_2^* = 0{\cdot}3$ W/m. deg (point a in Fig. 4) which decreases linearly with the heat flux decrease.

Such quantitative results shed light on one of the main reasons for instability of experimental installations reported in [4–8] for two-phase cooling of a homogeneous porous wall with water as a working fluid. In all of the installations the external heat flux density did not exceed 10^6 W/m^2, whereas the thermal conductivity of the porous metal and ceramic plates used there was never less than 10 W/m.deg. It should also be noted that the experiments were carried out under atmospheric [4–7] or even reduced [8] external pressure. The drop of the external pressure (Fig. 5) still greater reduces the boundary value of the vapour section thermal conductivity.

Such severe limitations are imposed upon the parameters of a stable system even for evaporation on the external surface $l = 1 \cdot 0$, when the accepted model has the highest accuracy. What is the physical nature of such severe limitations? It becomes obvious if expression (4) is written for water evaporation on the external surface

$$\lambda_2^* = \frac{q^* \delta c''}{i''_{e=1} - c_m T_\infty} \cdot \frac{1}{\left.\frac{1}{g}\frac{\mathrm{d}g}{\mathrm{d}l}\right|_{l=1}}. \quad (27)$$

A sharp decrease in the coolant flow rate with recession of the evaporation region inside the wall is great

$$\left.\frac{1}{g}\frac{\mathrm{d}g}{\mathrm{d}l}\right|_{l=1}$$

and presents one of the main reasons for instability of two-phase cooling of a homogeneous wall. Thus, a small shift of the phase transition region inside the plate causes a substantial decrease in the flow rate. The heat quantity supplied to this region and necessary for coolant heating and evaporation should reduce with the same rate. The external heat flux being constant, this holds provided that all the produced heat excess is absorbed by substantially reduced vapour flow rate, for which the vapour section of the porous wall should have a relevant very small thermal conductivity.

ANALYTICAL DETERMINATION OF THE RELIABILITY REGION OF THE TWO-PHASE TRANSPIRATION COOLING SYSTEM

The stability of operation is the most important but not the only requirement to the cooling system. It is also necessary that the external wall surface temperature does not exceed the permissible limit value T_1^{**} for reliable (without destruction) work of the porous material. Let us call this condition the reliability condition.

With the view of the above analysis it is natural to construct the reliability region in the same plane, $\lambda_2 - q$, because the cooling system remains stable and the external surface temperature does not exceed the permissible limit only in case of parameters variation inside the common area of the stability and reliability regions (stability and reliability region).

Analytical relations for the calculation of the reliability boundary follow from the condition that in case of the equilibrium phase transition in a certain region with the coordinate l the external surface temperature inside the plate equals the limit T_1^{**}. The first relation between the parameters at the reliability boundary (marked with 2 asterisks) expresses the condition of phase transpiration equilibrium

$$q^{**} = (i''_e - c_m T_\infty) \exp\left[\frac{G\delta c''}{\lambda^{**}}(1-l)\right]. \quad (28)$$

The second relation follows from the equation of heat balance at the external wall surface. In the accepted model the coolant moves along the normal from the bulk fluid to the wall to be cooled. All the heat supplied to the external surface is therefore spent to increase its enthalpy from the initial value $c_m T_\infty$ for the liquid phase to the enthalpy $i(P_1, T_1^{**})$ of the escaping vapour overheated to the temperature T_1^{**}

$$q^{**} = G[i(P_1, T_1^{**}) - c_m T_\infty]. \quad (29)$$

The results of the solution to equations (28)–(29) for the parameters λ_2^{**}, q^{**} have the form

$$\lambda_2^{**} = \frac{\delta c'' \mu'}{\beta/\alpha} \frac{m}{2n}\left[-1 + \sqrt{\left(1 + 4Re\frac{n}{m^2}\right)}\right]\frac{1-l}{\ln S} \quad (30)$$

$$q^{**} = [i(P_1, T_1^{**}) - c_m T_\infty]\frac{\mu'}{\beta/\alpha}\frac{m}{2n} \times \left[-1 + \sqrt{\left(1 + 4Re\frac{n}{m^2}\right)}\right]. \quad (31)$$

It should be taken into account that the intermediate parameter

$$S = \frac{i(P_1, T^{**}) - c_m T_\infty}{i''_e - c_m T_\infty} \quad (32)$$

incorporates the enthalpy i''_e of the dry saturated vapour that should be calculated in accordance with expressions (14), (13), (16), (7)–(9).

From the brief parametric representation of the reliability boundary

$$\lambda_2^{**} = \lambda_2^{**}(l, Re, P_1, T_1^{**}, \beta/\alpha, \delta, \text{ kind of cooler}) \quad (33)$$

$$q^{**} = q^{**}(l, Re, P_1, T_1^{**}, \beta/\alpha, T_\infty, \text{kind of cooler}) \quad (34)$$

it follows that it is to be defined by the same set of parameters as the stability boundary and by the parameter T_1^{**}.

It is also of interest to note that for a water-cooled system, at fixed T_1^{**} the intermediate parameter S is constant because $i'' = \text{const.}$ That is why for any coolant flow pattern the shape of the reliability boundary does not depend on the Reynolds number that follows from the results of transformation of equations (30)–(31) to the form

$$\lambda_2^{**} = \frac{q^{**}\delta c''}{[i(P_1, T_1^{**}) - c_m T_\infty]}\frac{1-l}{\ln S}. \quad (35)$$

The expression for the reliability boundary is rather visual if the transpiration cooling criterion of the vapour section

$$K_2^{**} = \frac{G\delta c''}{\lambda_2^{**}} = \frac{\ln S}{1-l} \quad (36)$$

is used as a parameter.

To illustrate relations (30)–(31) we shall plot the reliability boundary when the coordinate l of the phase transition surface is changing and all the other parameters are fixed. To compare the calculation with the results for the stability region the constant parameters were left unchanged: water is a coolant; $T_\infty = 20°C$; $P_1 = 10\,\text{bar}$; $\delta = 5\,\text{mm}$; $\alpha = 3 \cdot 5 \times 10^{13} m^{-2}$; $\beta = 1 \cdot 2 \times 10^8 m^{-1}$; $\beta/\alpha = 3 \cdot 44 \times 10^{-6} m$.

In Fig. 6 curve 2 plots the reliability boundary with the Reynolds number $Re = 0 \cdot 1$ and the maximum temperature of the heated surface $T_1^{**} = 1000°C$. Relevant values of the evaporation region coordinate are marked by numerals along the curve.

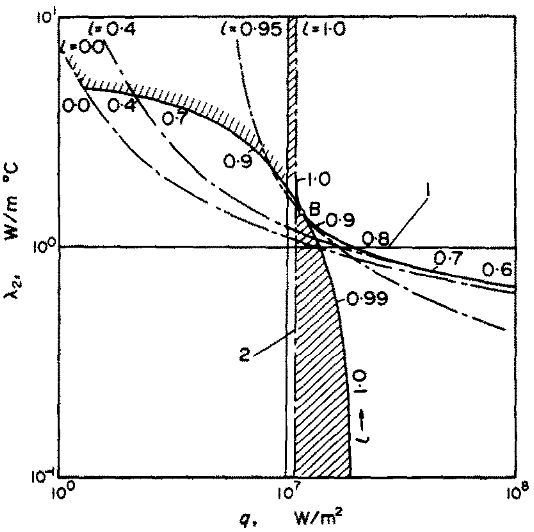

FIG. 6. Region of stable and reliable operation of equilibrium two-phase transpiration cooling system at $Re = 0{\cdot}1$: 1, stability boundary; 2, reliability boundary.

To define the reliability region we should construct dot-and-dash lines $l =$ const. of equilibrium phase transition. It turns out that the reliability condition holds for the systems whose parameters λ_2, q are presented by dots on dot–dash lines $l =$ const. upwards to the left from similar points of their intersection with the stability boundary

$$\lambda_2 > \lambda_2^{**}; \quad q < q^{**}. \tag{37}$$

The reliability region is shown by dashes and consists of two parts disposed at different sides of the reliability boundary. The boundary condition is plotted in the same figure as curve 1. The point of contact between the reliability and stability regions (point b) shows the place of transition of the reliability region from one of the sides of its boundary to the other. The common part of the stability and reliability regions where the conditions

$$\lambda_2^{**} < \lambda_2 < \lambda_2^{*} \quad q^{**} > q > q^{*} \tag{38}$$

are fulfilled, are shown as hatched regions in Fig. 6.

The comparison of the stability regions in Figs. 1 and 6 manifests that the reliability condition greatly narrows the region of permissible parameters. Phase transition is only possible in a very thin layer near the external surface. The width of the hatched area, that is the range of the permissible fluctuations of the heat flux increases generally with a decrease in the thermal conductivity of the vapour section, but the thermal conductivity in this case becomes extremely small.

Curve 2 in Fig. 3 is also a reliability boundary. The transition from the conditions with the dominating viscous drag to the inertial one has changed the form of contact of the stability and reliability boundaries. However, the conclusion about substantial narrowing of the range of permissible parameters does hold here.

The use of the family of stability and reliability regions relevant to different values of the Reynolds number makes it possible to determine not only permissible deviations of the external heat flux density and effective vapour thermal conductivity at constant pressure drop on the plate (constant Re number) from the calculated data, but also permissible pressure drop fluctuations.

The analytical investigation of the region of stable and reliable work of a two-phase transpiration cooling system has revealed one of the main reasons for instability of the experimental installations for two-phase water cooling of a homogeneous porous wall reported in literature.

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DETERMINATION DE LA REGION D'UTILISATION STABLE ET SURE DES SYSTEMES DE REFROIDISSEMENT PAR TRANSPIRATION EN EQUILIBRE BIPHASIQUE

Résumé—A l'aide des conditions de stabilité, en l'absence de brûlure homogène de la paroi, des expressions analytiques sont obtenues pour la région de variation des paramètres dans laquelle le système de refroidissement biphasique en milieu poreux est stable et la température de la paroi chauffée n'excède pas la limite pour un emploi convenable d'un matériau poreux. Il est trouvé que la stabilité du système dépend d'un nombre important de paramètres, le genre du réfrigérant étant l'un d'entre eux.

La condition de stabilité impose des limitations très strictes sur les caractéristiques du système dont on tient compte dans le cas d'un retrait de la région d'évaporation de la surface externe dans la plaque par une diminution importante de la vitesse d'écoulement du réfrigérant. Il apparait clairement de la comparaison des résultats obtenus avec les paramètres des installations expérimentales décrites dans les publications, que le fait de négliger les conditions ci-dessus, constitue en vérité, l'une des principales raisons de l'instabilité du refroidissement par transpiration biphasique d'une paroi homogêne avec refroidissement à l'eau.

BESTIMMUNG DES GEBIETS DES STABILEN UND SICH ZUVERLÄSSIG EINSTELLENDEN VORGANGS DER GLEICHGEWICHTS-ZWEIPHASEN-TRANSPIRATIONSKÜHLUNG

Zusammenfassung—Unter Verwendung der Stabilitätsbedingungen werden für das Gebiet, in dem das System der Zweiphasen-Transpirationskühlung stabil ist und die Temperatur der beheizten Oberfläche den Bereich für die zuverlässige Verwendung von porösem Material nicht überschreitet, analytische Beziehungen abgeleitet.

Es wurde festgestellt, daß die Stabilität des Systems von einer beträchtlichen Anzahl von Parametern, u. a. der Art des Kühlmittels, abhängt.

Die Stabilitätskriterien bedingen sehr enge Begrenzungen der Charakteristiken des Systems. Es wurde eine starke Abnahme des Kühlmittelstroms im Fall eines Rückgangs des Verdampfungsbereichs von der äußeren Oberfläche in die Platte hinein festgestellt. Der Vergleich der Ergebnisse aus anderen Experimenten, die der Literatur entnommen wurden, zeigt deutlich, daß eine Vernachlässigung der obigen Bedingungen in der Tat eine der wichtigsten Ursachen für die Instabilität der Zweiphasen-Transpirationskühlung einer homogenen Wand unter Benützung von Wasser als Kühlmittel ist.

ОПРЕДЕЛЕНИЕ ОБЛАСТИ ПАРАМЕТРОВ УСТОЙЧИВОЙ И НАДЕЖНОЙ РАБОТЫ РАВНОВЕСНОЙ СИСТЕМЫ ДВУХФАЗНОГО ПОРИСТОГО ОХЛАЖДЕНИЯ

Аннотация — С использованием условий устойчивости и отсутствия прогара однородной стенки выведены аналитические выражения для вычисления области параметров, при изменении внутри которой система двухфазного пористого охлаждения устойчива, а температура нагреваемой поверхности не превышает предельной для надежной экспуатации пористого материала. Установлено, что устойчивость системы определяется существенным количеством параметров, одним из которых является вид охладителя.

Условие устойчивости накладывает очень жесткие ограничения на характеристики системы, что объясняется резким уменьшением расхода охладителя при заглублении зоны испарения с внешней поверхности внутрь пластины. Из сравнения полученных результатов с парамеррами описанных в литературе экспериментальных установок становится очевидным, что несоблюдение указанных требований и является одной из основных причин неустойчивости двухфазного пористого охлаждения однородной стенки при использовании воды в качестве охладителя.